the ergonomics of `PL metatheory` in Lean

7 June 2024

the ergonomics of `PL metatheory` in Lean

or: I couldn’t convince people to use Agda for metatheory
but maybe I can convince you to try out Lean instead

this talk is about:

- tooling and editor support
- language features designed to take advantage of support
- demonstrating this with a real PL metatheory codebase

this talk is not about:

- how to do metatheory in Lean
- Lean’s type theory
- (okay maybe a little at the end. but I promise no typing rules)

Coq and Lean are:

- interactive proof assistants
- based on dependent types (proofs are terms)
- with automation via tactics

my issues with Coq

- P1: proof **reading** is missing ephemeral information
- P2: term **writing** isn’t incremental
- P3: proof **editing** is sequential
- P4: information for **learning** isn’t well integrated

specifically using:

- Coq-LSP
- also applies to VSCoq 1/CoqIDE
- (I don’t use emacs so idk Proof General)

P1: proof reading is missing ephemeral information

- Q1: what changes did this tactic make?
  - in `tac1; tac2`, what changes did tac1 make?
- Q2: what is the type of this subexpression?
  - what is the *context* of this subexpression?
- Q3: what case is this subproof for?

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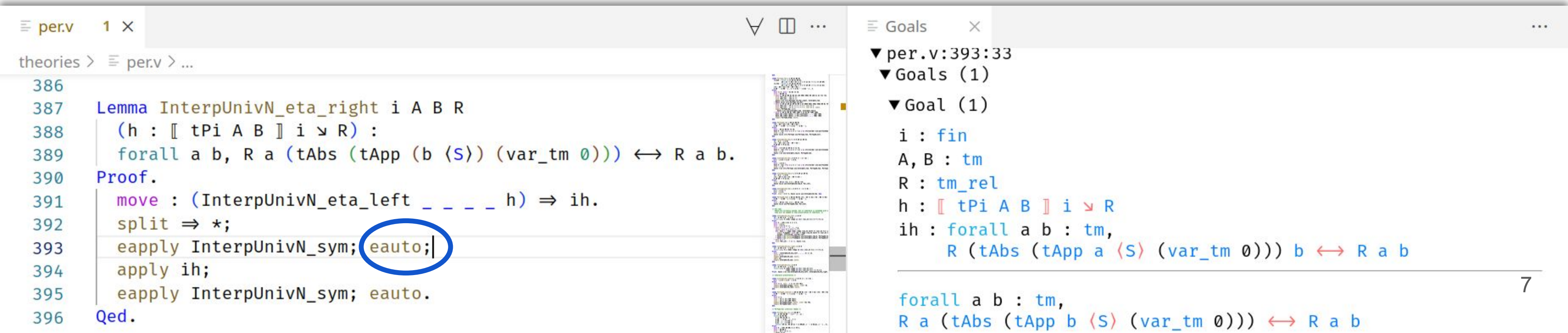

```
Lemma InterpUnivN_eta_right i A B R
  (h : ⟦ tPi A B ⟧ i ↘ R) :
  forall a b, R a (tAbs (tApp (b ⟨S⟩) (var_tm 0))) ⟷ R a b.
Proof.
  move : (InterpUnivN_eta_left _ _ _ _ h) ⇒ ih.
  split ⇒ *;
  eapply InterpUnivN_sym; eauto;
  apply ih;
  eapply InterpUnivN_sym; eauto.
Qed.
```

```
▼per.v:393:33
 ▼Goals (1)
  ▼Goal (1)
   i : fin
   A, B : tm
   R : tm_rel
   h : ⟦ tPi A B ⟧ i ↘ R
   ih : forall a b : tm,
        R (tAbs (tApp a ⟨S⟩ (var_tm 0))) b ⟷ R a b
   ──────────────────────────────────────────
   forall a b : tm,
   R a (tAbs (tApp b ⟨S⟩ (var_tm 0))) ⟷ R a b
```

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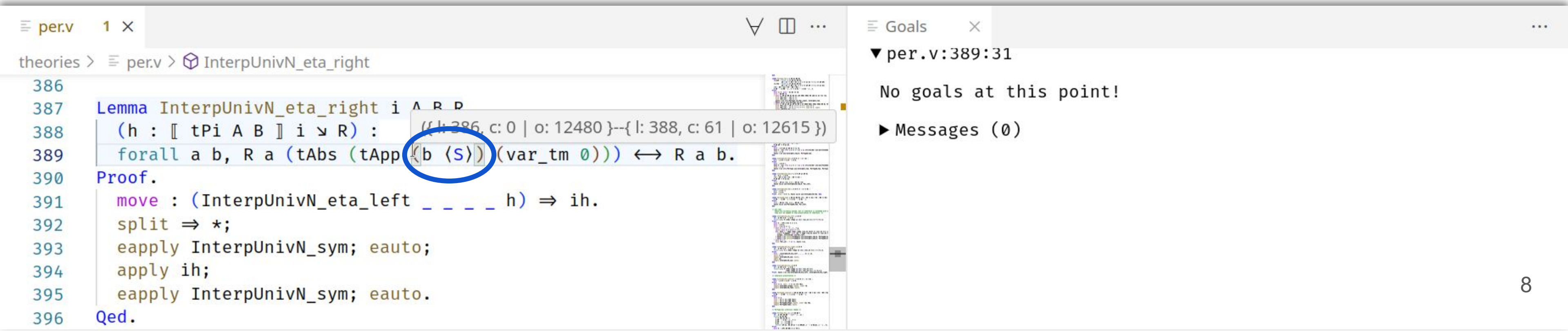

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  apply ih;
  eapply InterpUnivN_sym; eauto.
Qed.
```

({ l: 386, c: 0 | o: 12480 }--{ l: 388, c: 61 | o: 12615 })

No goals at this point!

P2: term writing isn’t incremental

- Q1: how do I fill in this term later?
- Q2: how do I fill in this term with tactics?
    - if I don’t know the exact tactic I need?
    - if I need multiple tactics/generate subgoals?

P2: term writing isn't incremental

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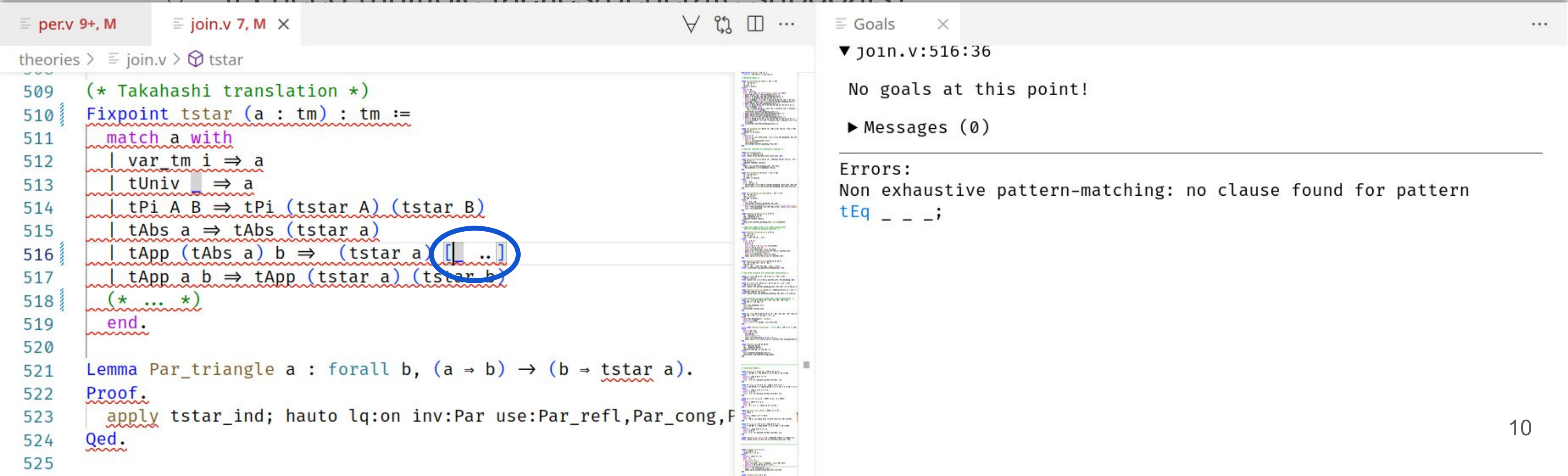

```
(* Takahashi translation *)
Fixpoint tstar (a : tm) : tm :=
  match a with
  | var_tm i ⇒ a
  | tUniv _ ⇒ a
  | tPi A B ⇒ tPi (tstar A) (tstar B)
  | tAbs a ⇒ tAbs (tstar a)
  | tApp (tAbs a) b ⇒  (tstar a) [_ ..]
  | tApp a b ⇒ tApp (tstar a) (tstar b)
  (* ... *)
  end.

Lemma Par_triangle a : forall b, (a ⇒ b) → (b ⇒ tstar a).
Proof.
  apply tstar_ind; hauto lq:on inv:Par use:Par_refl,Par_cong,P
Qed.
```

```
▼ join.v:516:36

 No goals at this point!

 ▶ Messages (0)

Errors:
Non exhaustive pattern-matching: no clause found for pattern
tEq _ _ _;
```

P2: term writing isn't incremental

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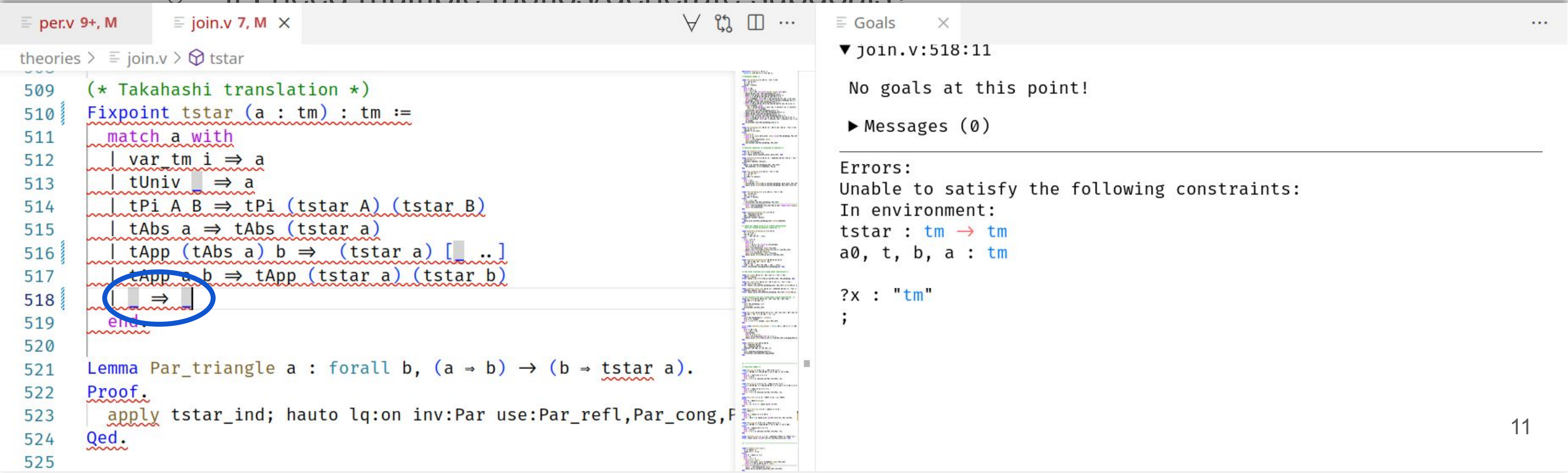

```
(* Takahashi translation *)
Fixpoint tstar (a : tm) : tm :=
  match a with
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  | tUniv _ ⇒ a
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  | tApp (tAbs a) b ⇒  (tstar a) [_ ..]
  | tApp a b ⇒ tApp (tstar a) (tstar b)
  | _ ⇒ _
  end

Lemma Par_triangle a : forall b, (a ⇒ b) → (b ⇒ tstar a).
Proof.
  apply tstar_ind; hauto lq:on inv:Par use:Par_refl,Par_cong,P
Qed.
```

```
▼ join.v:518:11

 No goals at this point!

 ▶ Messages (0)

Errors:
Unable to satisfy the following constraints:
In environment:
tstar : tm → tm
a0, t, b, a : tm

?x : "tm"
;
```

P2: term writing isn't incremental

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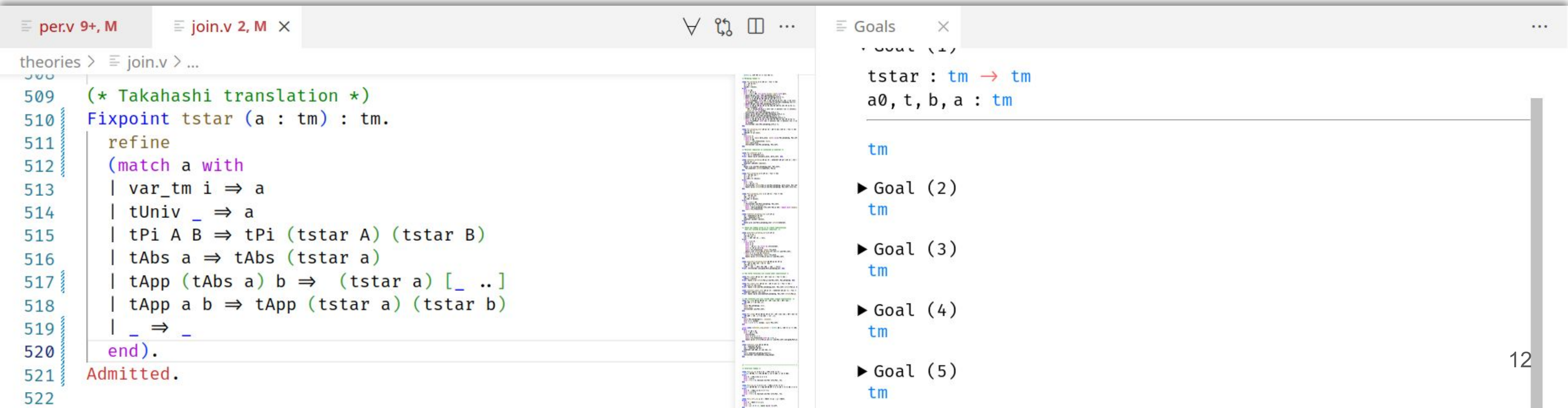

```
(* Takahashi translation *)
Fixpoint tstar (a : tm) : tm.
  refine
  (match a with
  | var_tm i ⇒ a
  | tUniv _ ⇒ a
  | tPi A B ⇒ tPi (tstar A) (tstar B)
  | tAbs a ⇒ tAbs (tstar a)
  | tApp (tAbs a) b ⇒  (tstar a) [_ ..]
  | tApp a b ⇒ tApp (tstar a) (tstar b)
  | _ ⇒ _
  end).
Admitted.
```

```
tstar : tm → tm
a0, t, b, a : tm
______________________
tm

▶ Goal (2)
tm

▶ Goal (3)
tm

▶ Goal (4)
tm

▶ Goal (5)
tm
```

P3: proof editing is (more or less) sequential

- Q1: when I change a definition, what proofs are broken?
  - did the proofs break in an expected way? or did I change the wrong definition?
  - how much work is left? what should I repair next?
- Q2: when I change a definition, what proof *cases* are broken?
  - am I missing a new case?
  - is an old case no longer covered by automation?

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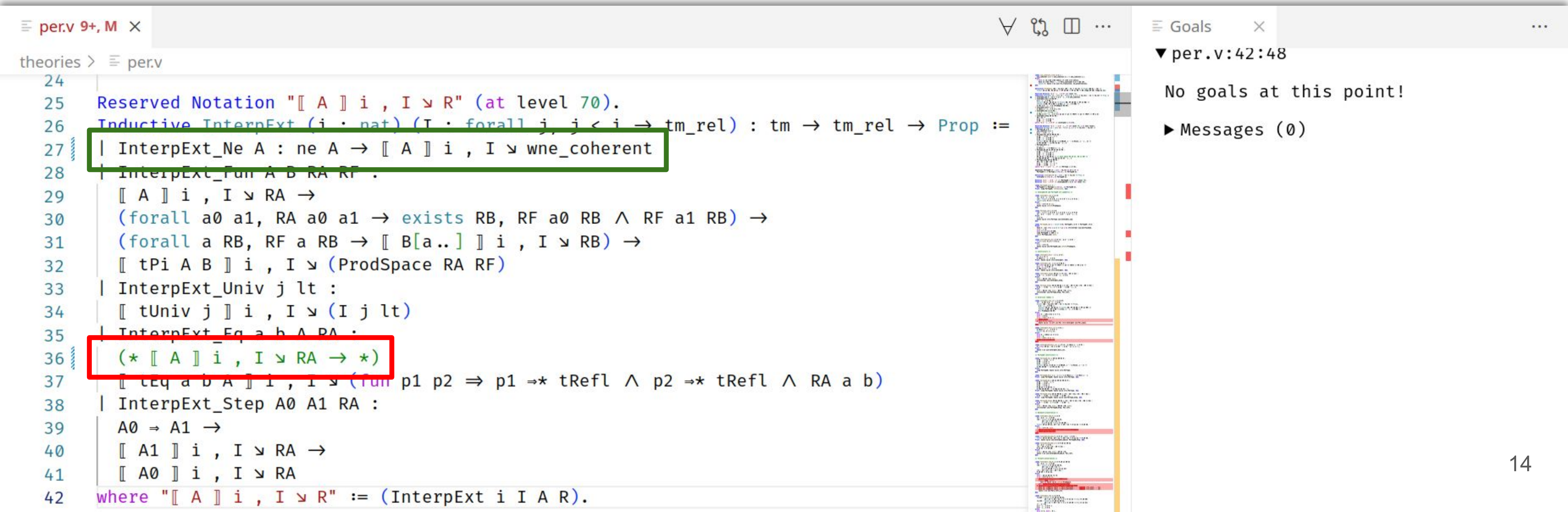

```
Reserved Notation "⟦ A ⟧ i , I ↘ R" (at level 70).
Inductive InterpExt (i : nat) (I : forall j, j < i → tm_rel) : tm → tm_rel → Prop :=
| InterpExt_Ne A : ne A → ⟦ A ⟧ i , I ↘ wne_coherent
| InterpExt_Fun A B RA RF :
  ⟦ A ⟧ i , I ↘ RA →
  (forall a0 a1, RA a0 a1 → exists RB, RF a0 RB ∧ RF a1 RB) →
  (forall a RB, RF a RB → ⟦ B[a..] ⟧ i , I ↘ RB) →
  ⟦ tPi A B ⟧ i , I ↘ (ProdSpace RA RF)
| InterpExt_Univ j lt :
  ⟦ tUniv j ⟧ i , I ↘ (I j lt)
| InterpExt_Eq a b A RA :
  (* ⟦ A ⟧ i , I ↘ RA → *)
  ⟦ tEq a b A ⟧ i , I ↘ (fun p1 p2 ⇒ p1 ⇒* tRefl ∧ p2 ⇒* tRefl ∧ RA a b)
| InterpExt_Step A0 A1 RA :
  A0 ⇒ A1 →
  ⟦ A1 ⟧ i , I ↘ RA →
  ⟦ A0 ⟧ i , I ↘ RA
where "⟦ A ⟧ i , I ↘ R" := (InterpExt i I A R).
```

```
▼ per.v:42:48

No goals at this point!

▶ Messages (0)
```

P3: proof editing is (more or less) sequential

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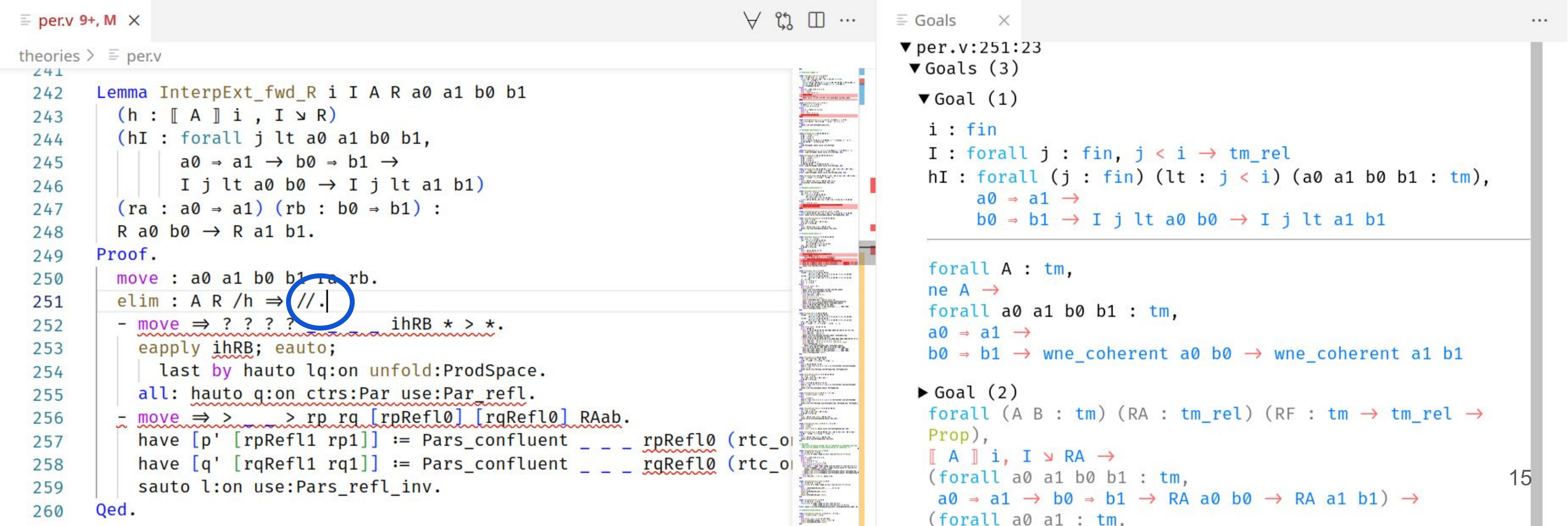

```
Lemma InterpExt_fwd_R i I A R a0 a1 b0 b1
  (h : ⟦ A ⟧ i , I ↘ R)
  (hI : forall j lt a0 a1 b0 b1,
        a0 ⇒ a1 → b0 ⇒ b1 →
        I j lt a0 b0 → I j lt a1 b1)
  (ra : a0 ⇒ a1) (rb : b0 ⇒ b1) :
  R a0 b0 → R a1 b1.
Proof.
  move : a0 a1 b0 b1 ra rb.
  elim : A R /h ⇒ //.
  - move ⇒ ? ? ? ? _ _ _ _ ihRB * > *.
    eapply ihRB; eauto;
      last by hauto lq:on unfold:ProdSpace.
    all: hauto q:on ctrs:Par use:Par_refl.
  - move ⇒ > _ _ _ > rp rq [rpRefl0] [rqRefl0] RAab.
    have [p' [rpRefl1 rp1]] := Pars_confluent _ _ _ rpRefl0 (rtc_o
    have [q' [rqRefl1 rq1]] := Pars_confluent _ _ _ rqRefl0 (rtc_o
    sauto l:on use:Pars_refl_inv.
Qed.
```

```
▼ per.v:251:23
 ▼ Goals (3)
  ▼ Goal (1)
   i : fin
   I : forall j : fin, j < i → tm_rel
   hI : forall (j : fin) (lt : j < i) (a0 a1 b0 b1 : tm),
        a0 ⇒ a1 →
        b0 ⇒ b1 → I j lt a0 b0 → I j lt a1 b1
   ______________________________________________
   forall A : tm,
   ne A →
   forall a0 a1 b0 b1 : tm,
   a0 ⇒ a1 →
   b0 ⇒ b1 → wne_coherent a0 b0 → wne_coherent a1 b1

  ▶ Goal (2)
   forall (A B : tm) (RA : tm_rel) (RF : tm → tm_rel →
   Prop),
   ⟦ A ⟧ i, I ↘ RA →
   (forall a0 a1 b0 b1 : tm,
    a0 ⇒ a1 → b0 ⇒ b1 → RA a0 b0 → RA a1 b1) →
   (forall a0 a1 : tm,
```

P4: information isn’t well integrated

- Q1: what does this tactic do?
- Q2: how do I use this syntax?
  - let’s see *you* try to remember full syntax for `match` off the top of your head

claim: Lean’s tooling & design help solve these problems

P4: information isn’t well integrated

- Q1: what does this tactic do?
- Q2: how do I use this syntax?

  A: documentation on hover!

- bonus: cross-file ctrl-click navigation

P1: proof reading is missing ephemeral information

- Q1: what changes did this tactic make?
  - in `tac1; tac2`, what changes did tac1 make?
- Q2: what is the type of this subexpression?
  - what is the *context* of this subexpression?

A: infoview at cursor!

- Q3: what case is this subproof for?

A3: *tagged cases* help retain this info

P2: term writing isn’t incremental

- Q1: how do I fill in this term later?
  A1: with *typed holes*
- Q2: how do I fill in this term with tactics?
  A2: `by` dropping into *proof mode, anywhere*

P3: proof editing is (more or less) sequential

- Q1: when I change a definition, what proofs are broken?
    - did the proofs break in an expected way? or did I change the wrong definition?
    - how much work is left? what should I repair next?
- Q2: when I change a definition, what proof *cases* are broken?
    - am I missing a new case?
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A: [demo]

tactics I enjoy

- `split`: pushes a goal inside of a `match`
- `calc`: explicit equational reasoning
- `apply_rules [lem, …]`: keep applying lemmas and hypotheses without backtracking (like Coq's `eauto … using …`)

other tools and libraries

for Coq

- SSReflect
- CoqHammer
- Mathcomp
- Psatz

for Lean

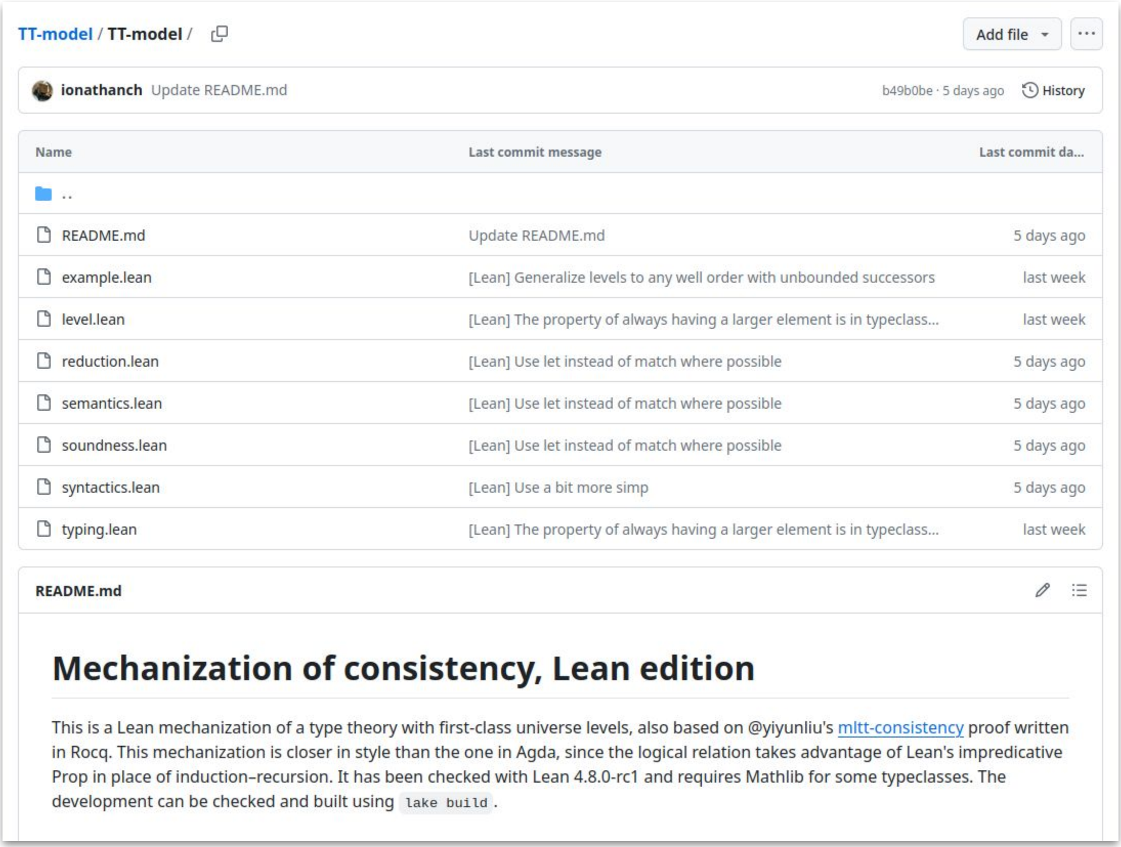
- LeanSSR
  https://github.com/verse-lab/lean-ssr
- Aesop
  https://github.com/leanprover-community/aesop
- Mathlib
  https://github.com/leanprover-community/mathlib4
- `(n)linarith` (part of Mathlib)

my issues with Lean

basically designed specifically targeting mathematics

- no structural recursion on inductive predicates
  - inductive *propositions* are fine. smth abt decreasing measures
  - pretty sure this is a bug
- no induction tactic on mutual inductives
  - encode mutual inductives as single inductives
  - Coq doesn’t have great mutual inductive support anyway
- no coinduction
  - RIP itrees folks ig
- no modules
  - use typeclasses
- no Agda-style unification
  - :(((

ported existing development in ~1 week



ionathanch Update README.md b49b0be · 5 days ago

Name	Last commit message	Last commit da...
..		
README.md	Update README.md	5 days ago
example.lean	[Lean] Generalize levels to any well order with unbounded successors	last week
level.lean	[Lean] The property of always having a larger element is in typeclass...	last week
reduction.lean	[Lean] Use let instead of match where possible	5 days ago
semantics.lean	[Lean] Use let instead of match where possible	5 days ago
soundness.lean	[Lean] Use let instead of match where possible	5 days ago
syntactics.lean	[Lean] Use a bit more simp	5 days ago
typing.lean	[Lean] The property of always having a larger element is in typeclass...	last week

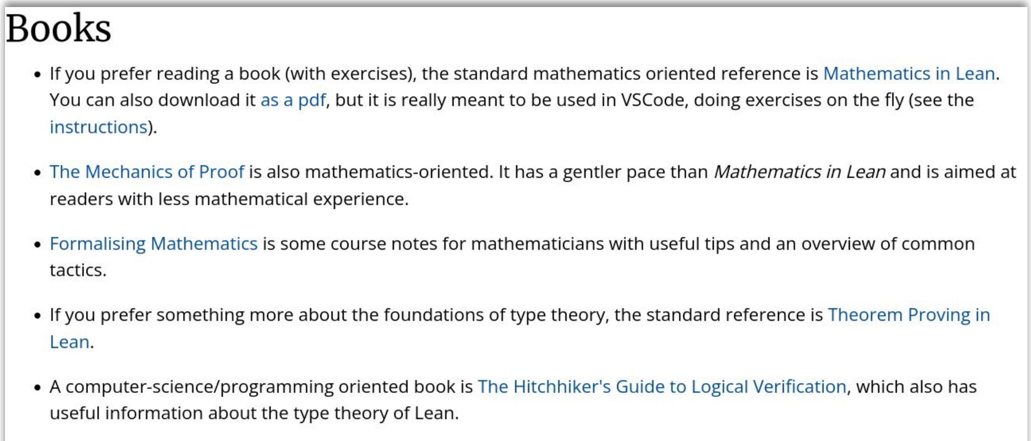
README.md

Mechanization of consistency, Lean edition

This is a Lean mechanization of a type theory with first-class universe levels, also based on @yiyunliu's mltt-consistency proof written in Rocq. This mechanization is closer in style than the one in Agda, since the logical relation takes advantage of Lean's impredicative Prop in place of induction–recursion. It has been checked with Lean 4.8.0-rc1 and requires Mathlib for some typeclasses. The development can be checked and built using `lake build`.

learning Lean

- Zulip (to ask questions): https://leanprover.zulipchat.com/
- Lean in your browser: https://live.lean-lang.org/
- reference manual: https://lean-lang.org/lean4/doc/
- library docs: https://leanprover-community.github.io/mathlib4_docs/
- learning resources: https://leanprover-community.github.io/learn.html



Books

- If you prefer reading a book (with exercises), the standard mathematics oriented reference is Mathematics in Lean. You can also download it as a pdf, but it is really meant to be used in VSCode, doing exercises on the fly (see the instructions).
- The Mechanics of Proof is also mathematics-oriented. It has a gentler pace than *Mathematics in Lean* and is aimed at readers with less mathematical experience.
- Formalising Mathematics is some course notes for mathematicians with useful tips and an overview of common tactics.
- If you prefer something more about the foundations of type theory, the standard reference is Theorem Proving in Lean.
- A computer-science/programming oriented book is The Hitchhiker's Guide to Logical Verification, which also has useful information about the type theory of Lean.